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Particularity of Fluid Flow in Highly Heterogeneous Stochastic Porous Media

Posvyanskii D.V.

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Abstract

Reservoir permeability is a critical parameter for oil production estimation, exhibiting high spatial variability which suggests treatment as a random function. The present paper focuses on the study of steady state flow of a single phase fluid in a two-dimensional heterogeneous formation with random permeability characterized by a log-normal distribution with coefficient of variation γ_k and correlation length λ . The aim of this study is to determine the relationship between the flux variance σ_q^2 and γ_k . Previously available analytical solutions to this problem have primarily relied on perturbation-based methods, which are approximate and applicable for relatively small values of γ_k . This study employs the Monte Carlo method to investigate fluid flow in reservoirs with stochastic permeability where γ_k ranges from 0.1 to 4.0. The results show that σ_q^2 exhibits a maximum at a certain critical value of γ_k , denoted as γ_c . For highly heterogeneous formations $\gamma_k > \gamma_c$, this study shows that the fluid flux follows the Poisson distribution and flux variance is proportional to its mean value $\langle q \rangle$. Such behavior is characteristic of random variables that follow a Poisson distribution. It is well known that the Poisson distribution is a discrete probability distribution and this study shows that for the case $\gamma_k > \gamma_c$, a network of discrete preferential flow channels forms. Such channels are not associated with any extended geological objects like faults or fractures and are caused by a random medium. The possibility of detecting such channels is assessed through the analysis of pressure build-up curves. The process of water-oil displacement is also studied for the case of highly heterogeneous reservoirs. Water breakthrough time shown as a function of γ_k and λ illustrates the noticeable influence of the statistical characteristics of permeability on the oil displacement process, that can be used in history matching process.

Keywords Stochastic permeability · Shot noise · Well inflow

1 Introduction

Reservoir permeability is an essential reservoir engineering parameter which greatly impacts well productivity. Geological formations exhibit significant spatial variability and permeability is one of the most spatially variable petrophysical properties. Since measurements of local permeability values are typically scarce, it can be considered as a random function [1, 2]. In turn, the randomness of formation permeability propagates to the flow-related variables such as well production rate or bottom hole pressure, which also become random. That leads to uncertainty in production prediction and increases risks in field development.

In [3, 4] it was shown that permeability measurements on core samples follow a log-normal distribution, which is commonly employed in modeling flow through random media [2, 5]. This study assumes that permeability is log-normally distributed, characterized by a mean value $\langle k \rangle$, a variance σ_k^2 , and a correlation length λ , that



describes its spatial variability. Subsequently, the coefficient of variation $\gamma_k = \sigma_k / \langle k \rangle$ will be used to describe the amplitude of permeability fluctuations.

Flow in a porous media with a random permeability has been a subject of interest for many years and various approaches have been proposed to solve this problem.

Monte Carlo simulation (MCS) is the most popular tool for describing flow in heterogeneous media. This straightforward method can solve a wide range of problems and it can be considered as a reference approach. However, to achieve statistical convergence and accuracy level, MCS requires many simulation runs, which makes it computationally expensive.

The other framework is a perturbation-based stochastic flow simulation in which formation heterogeneity is treated as a perturbation. The solution of the governed flow equations is expanded in a series of powers of perturbation. Typically, this series is truncated at the second term of the expansion and therefore this method accounts only for small permeability fluctuations ($\gamma_k \ll 1$). Studies by [2] and [5] applied the perturbation method to obtain approximate expressions for well rate and bottom hole variances, retaining linear terms of σ_k^2 . In [6] and [7], the well inflow in stochastic reservoirs was studied using the diagrammatic approach, which graphically represents the perturbation series and enables the summation of significant terms. However, the diagram technique is limited to rather simple geological situations and, for more realistic cases, requires the computation of complex multidimensional integrals.

The approach of statistical moment equations (SME) can be employed to describe liquid flow in random porous media. The low-order SME approximation (LOSME) is used for solving practical problems [8]. In this approach, higher-order terms are truncated in the equations for the first and second moments of pressure. Based on perturbation expansion, LOSME simplifies handling various boundary conditions, complex flow geometries, and reservoir shapes. Li et al. [9] present a comparison of MCS and LOSME results and they demonstrate that low-order approximation provides robust estimates of pressure mean values even at high values of γ_k . Zhang and Winter [8] show good agreement between LOSME and MCS results for the second pressure moment and they also calculate flux variance.

This study presents a numerical investigation of single-phase fluid flow in a medium characterized by random permeability. The formations under consideration are those where γ_k and the correlation length vary over a wide range. Quantification of flow uncertainty is the main subject of this analysis and the ultimate goal is to find the relationship between flux variance and the coefficient of variation of permeability γ_k for different values of λ . The obtained results are compared with existing analytical expressions for flux variance, derived within the framework of perturbation theory. It is shown that at high values of variance, the random flux follows the Poisson statistic. Such behavior is largely similar to the phenomenon of shot noise, which is observed in electronic devices. Using the theory of shot noise [10], as employed in radio physics, one can derive expression for the variance of single-phase flux. The obtained analytical estimates are in good agreement with the results of numerical modelling. It can be expected that the formation of discrete flow channels will also affect the flow of the two-phase fluid and the oil displacement process. This study examines how the statistical characteristics of the permeability field influence the water breakthrough time in the well.

2 Mathematical model

The mathematical model of a single phase steady-state fluid flow in a porous medium is based on Darcy's law, which is a linear relationship between the fluid velocity and the pressure gradient

$$\vec{v}(\mathbf{r}) = -\frac{\mathbf{k}(\mathbf{r})}{\mu} \nabla p(\mathbf{r}) \quad (1)$$

and the equation of continuity,

$$\nabla \cdot \vec{v}(\mathbf{r}) = 0 \quad (2)$$

where μ is the viscosity of the fluid and $\mathbf{k}(\mathbf{r})$ is the permeability tensor which is a function of position $\mathbf{r} = (x, y, z)$. The governing equation for fluid pressure is obtained by combining Eqs. 1 and 2

$$\nabla \cdot (\mathbf{k}(\mathbf{r}) \nabla p(\mathbf{r})) = 0 \quad (3)$$

This Eq. 3 should be solved subject to boundary conditions, which depend on the specific task and will be defined later.

The two point covariance function $C_{kk}(\mathbf{r}, \mathbf{r}')$ of the permeability field depends only on the distance between points and it can take various forms. In the present work, the following one will be used

$$C_{kk}(\mathbf{r}, \mathbf{r}') = \sigma_k^2 e^{-\frac{(\mathbf{r} - \mathbf{r}')^2}{\lambda^2}} \quad (4)$$

The randomness in permeability propagates in reservoir pressure and the flux vector which are also represented as random functions. The aim of this work is to determine the dependence of flux variance on the normalized standard deviation of permeability γ_k . The most straightforward method for solving Eq. 3 with stochastic permeability is Monte Carlo simulation. Within this approach, various realizations of a random field are generated and for each of them Eq. 3 should be solved. The flux mean value $\langle q \rangle$ and its variance σ_q^2 can be calculated using the following expressions

$$\langle q \rangle = \sum_i^N q_i / N \quad \sigma_q^2 = \sum_i^N (q_i - \langle q \rangle)^2 / N \quad (5)$$

where index i denotes the realization number, and N is the total number of realizations. q_i , the flux for the i -th realization depends on the specific problem being solved, and the method for its calculation will be described below. As N tends to infinity, expressions converges to the mean and the variance. The number N must be large enough to ensure the stability of the result obtained and may depend on the task being considered.

The permeability realizations for the given $\langle k \rangle$, γ_k and λ are generated using two methods: the covariance matrix decomposition method and the Fast Fourier Transformation (FFT) method. The FFT method implementation used was the one available in a commercial software package. It was verified that the method of random field generation has virtually no effect on the results of the calculations of $\langle q \rangle$ and σ_q^2 .

3 Uniform flow in stochastic porous media

In this section a box-shaped flow domain of sizes s_x, s_y, s_z is considered, where flow is induced by a uniform pressure drop between two opposite boundaries. Assuming that the pressure does not vary in the z -direction, the problem simplifies to a two-dimensional formulation. For clarity, it is assumed that the pressure drop is applied along x -axis

$$\Delta p = p(0, y) - p(s_x, y) = \text{const} \quad (6)$$

and no-flow condition are imposed at another two boundaries

$$\frac{\partial p(x, 0)}{\partial y} = 0 \quad \frac{\partial p(x, s_y)}{\partial y} = 0 \quad (7)$$

In this case, Eq. 3 has the following form

$$\frac{\partial}{\partial x} \left(k(x, y) \frac{\partial p(x, y)}{\partial x} \right) + \frac{\partial}{\partial y} \left(k(x, y) \frac{\partial p(x, y)}{\partial y} \right) = 0 \quad (8)$$

Table 1 Parameters of the model

Drainage size $s_x = s_y$	8000	m
Thickness s_z	1	m
Mean permeability	50	mD
Porosity	0.1	
Viscosity	1	cP
Lateral cell size $\Delta x = \Delta y$	50	m
Vertical cell size Δz	1	m

subject to boundary conditions Eqs. 6 and 7. Equation 8 is solved for each permeability realization and the flux through the transverse boundary of the reservoir is given by

$$q_i = -\frac{s_z}{\mu} \int_0^{s_y} dy k_i(x, y) \frac{\partial p_i(x, y)}{\partial x} \quad (9)$$

Substituting Eq. 9 in Eq. 5 values $\langle q \rangle$ and σ_q^2 are calculated for the given γ_k . In the paper, calculations are carried out for the model, whose parameters are listed in the Table 1.

As pointed out in [11], for obtaining an accurate numerical solution the correlation length should exceed the size of the grid cells Δx by four times or more. All results presented below are obtained for $\lambda \geq 200 m$ and this condition is satisfied.

Figure 1 shows the dependence of the mean flux (left panel) and its variance (right panel) on the coefficient of variation γ_k . The mean flux and its variance are normalized with respect to q_0 and q_0^2 , respectively, where q_0 denotes the flux in a homogeneous medium with permeability $\langle k \rangle$ and it does not depend on γ_k . q_0 is given by the expression

$$q_0 = \frac{\langle k \rangle}{\mu} J s_y s_z$$

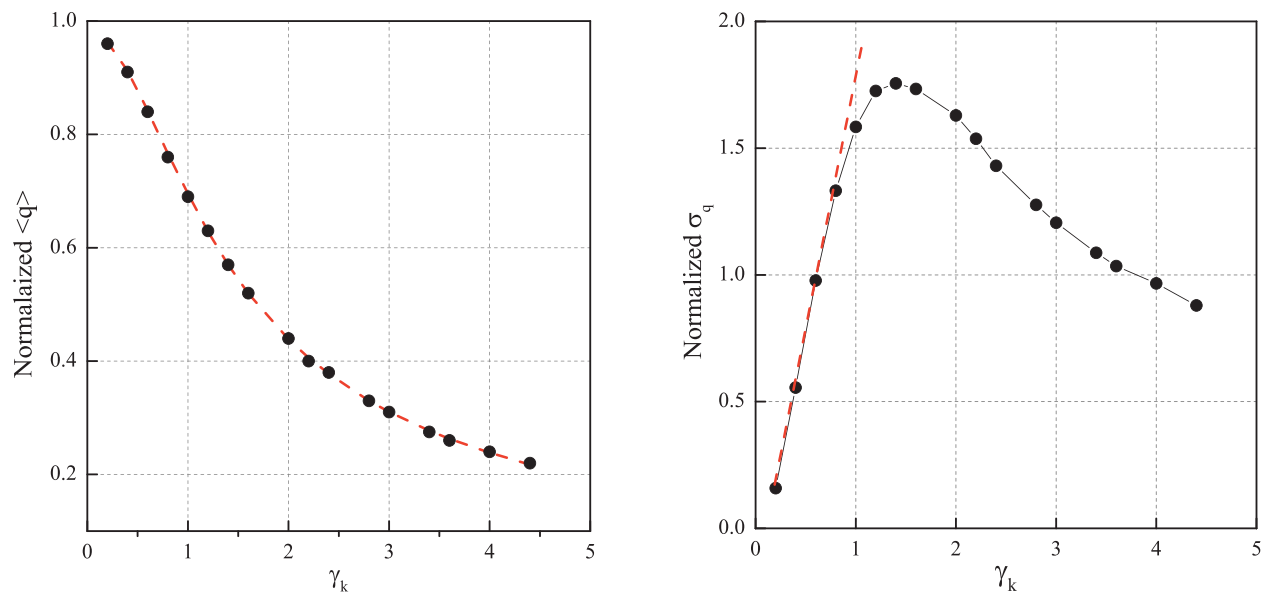


Fig. 1 Dependencies of the normalized mean flux $\langle q \rangle$ (left) and the standard deviation σ_q (right) on the coefficient variation of permeability γ_k . The red dashed lines represent the analytical relationships for the mean flux and variance presented in the literature. Normalization is performed using q_0 , which corresponds to the flux in a homogeneous reservoir with permeability $\langle k \rangle$

where J is the mean negative pressure gradient.

As indicated in the Table 1, the simulations were carried out using a grid with $\Delta x = \Delta y = 50 \text{ m}$. Reducing the grid cell size by half alters the calculated mean flux and variance by less than 1%. The calculation on the fine grid were performed for large values $\gamma_k > 2.0$.

The analytical dependence of $\langle q \rangle$ on γ_k is given by the following expression

$$\langle q \rangle = \frac{k_{eff}(\gamma_k)}{\langle k \rangle} q_0 \quad (10)$$

and for two-dimensional case

$$k_{eff} = k_g = \langle k \rangle e^{-\sigma_Y^2/2} \quad (11)$$

$$\sigma_Y^2 = \ln(1 + \gamma_k^2)$$

where $Y = \ln k$ and k_g denotes the geometric average of permeability. The expression for effective permeability was previously obtained from [5, 12–14]. The analytical expression given by Eq. 10, shown as a red dashed line in the left panel of Fig. 1, agrees well with the MCS results over the full range of γ_k .

Unlike the mean value, the flux variance is a non-monotonic function of γ_k and it has a maximum at a specific value γ_c . For $\gamma_k > \gamma_c$ the increase in γ_k leads to a decrease in σ_q^2 . For small permeability fluctuations $\gamma_k \ll 1$, the expression for dimensionless σ_q^2 was derived using perturbation theory. [2]. For $s_x \gg \lambda$, it can be simplified to

$$\sigma_q^2 = \pi \left(\frac{\lambda}{s_y} \right)^2 \gamma_k^2 \quad (12)$$

However, for large values of γ_k this expression does not even qualitatively coincide with MCS results.

Figure 2 illustrates the relationship between the flux variance (left plot) and the third moment (right plot) as functions of the mean flux. It can be seen that at lower values of $\langle q \rangle$ (that corresponds to large values of γ_k) this moments are linear functions of $\langle q \rangle$. Such behavior of the second and third moments may be a consequence

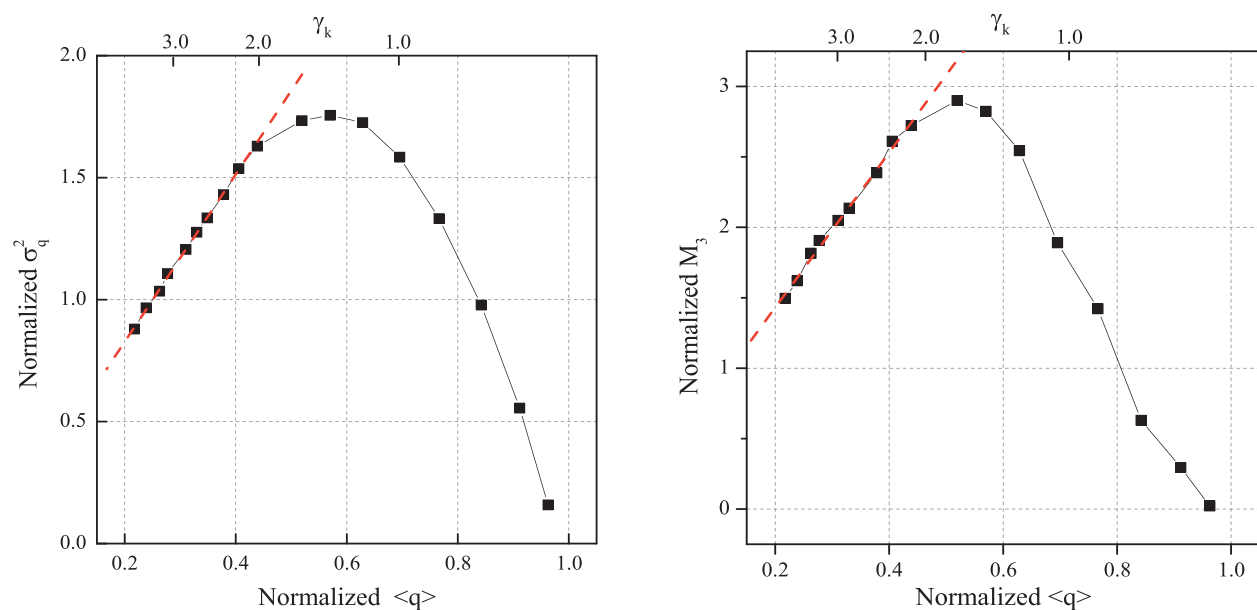
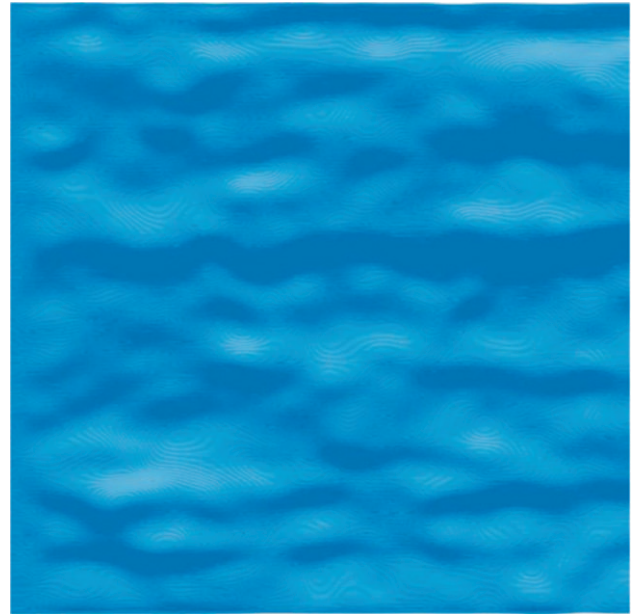


Fig. 2 The normalized second (left) and the third (right) flux moments as functions of mean flux. The red dashed lines show the linear fitting of these moments for $\gamma_k > \gamma_c$. Normalization is carried out with respect to q_0^2 and q_0^3 , respectively

Fig. 3 The spatial distribution of streamlines for small fluctuations of the permeability field $\gamma_k = 0.2$



of the random flow following the Poisson distribution, in which all cumulants are equal to the mean value and it is easy to show that all higher central moments proportional to the first one. Based on this, one can suggest that the random flux conforms the Poisson distribution when $\gamma_k > \gamma_c$. The similar phenomena are observed in electronic devices where the variance of electrical current proportional to its mean value [15]. It is called shot noise and it originates due to the discrete nature of electric charge and this analogy will be used in the following discussion. The Poisson distribution is a discrete type distribution that is often used to calculate the probability of rare events. This raises the question of the nature of discreteness in the problem of fluid flow in a porous medium. To answer this question, streamline realizations are constructed for various values of γ_k . In the case of weak reservoir heterogeneity $\gamma_k \ll \gamma_c$, the streamlines are uniformly distributed across the model domain, as shown in the Fig. 3.

Figure 4 depicts the spatial permeability realization for $\gamma_k = 3.0$ (the left plot) and the corresponding streamlines (the central plot). The combination of these two views is shown in the right plot. This figure illustrates that, for $\gamma > \gamma_c$, a discrete network of channels forms within the reservoir, through which fluid predominantly flow. These channels are self induced caused by the “focusing effect” of a random media [16]. The formation of a system of discrete channels leads to the appearance of the Poisson distribution.

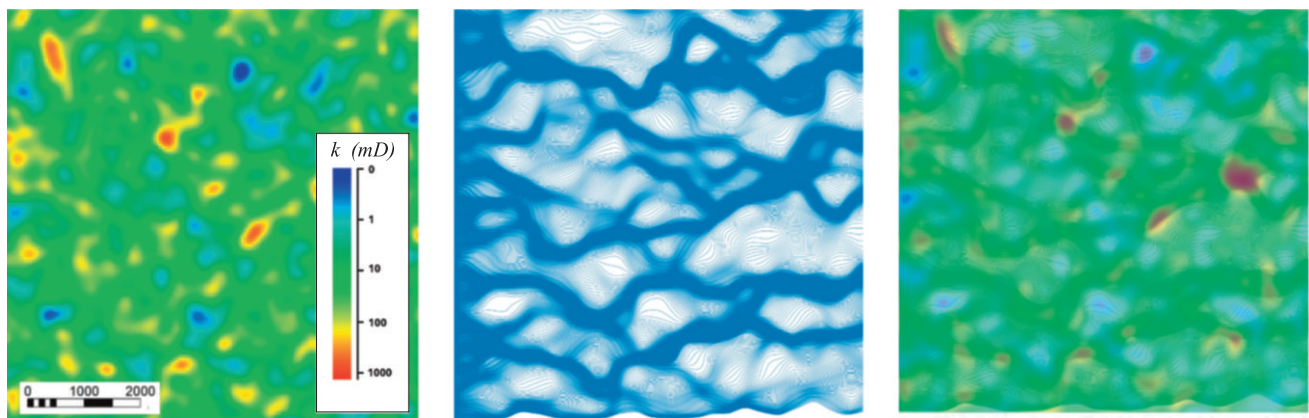


Fig. 4 The spatial permeability map for $\gamma_k = 3.0$ (left) and corresponding streamlines (center). The combination of these views (right)

In the framework of the LOSME approach, the spatial dependence of the variance of the longitudinal flux component $\sigma_{q_x}^2$ in an unbounded two-dimensional domain was obtained in [17] and is given by the following expression:

$$\sigma_{q_x}^2 = \frac{3}{8} \sigma_Y^2 k_g J \quad (13)$$

For the unbounded domain $\sigma_{q_x}^2$ does not depend on the coordinate.

Taking into account Eq. 11, it becomes clear that expression Eq. 13 does not yield a linear dependence of flux variance on the mean at higher values of γ_k . Apparently, this is related to the necessity of accounting for the contribution of higher-order pressure moments. In [18], numerical solution of LOSME were performed for a reservoir with a rectangular geometry, and the results demonstrate good agreement with Eq. 13.

Figure 5 shows the distribution of flux through the lateral surface at $x = s_x$ for various values of γ_k . The distribution corresponding to $\gamma_k = 0.2$ is shown in blue, and for $\gamma_k = 3.0$ in red. As can be seen, in the case of large permeability fluctuations, the flux can be represented as a series of random pulses. All pulses are mutually independent, with both their amplitudes and positions treated as random variables that follow a Poisson distribution [10].

The mean and variance of the random pulse process can be calculated in a manner similar to what is used in the shot noise theory, where signal is represented as a pulse train of the following form

$$\xi(t) = \sum a_i F(t - t_i), \quad (14)$$

where $F(t)$ is a deterministic function of time, which describes a shape of pulses. a_i and t_i represents the amplitude and the arrival time of the pulse correspondingly and both are independent random variables. The first two moments of a random pulse process are given by [15]

$$\begin{aligned} \langle \xi \rangle &= \nu \int da f(a) a \int_{-\infty}^{\infty} dx F(x) \\ \sigma_{\xi}^2 &= \nu \int da f(a) a^2 \int_{-\infty}^{\infty} dx F^2(x) \end{aligned} \quad (15)$$

Fig. 5 Flux distribution through the transverse side surface at $x = s_x$. The distribution for $\gamma_k = 0.2$ is shown in blue, while the distribution for $\gamma_k = 3.0$ is shown in red. Coordinate is normalized by the size s_y , while the flux is normalized by the total flux through the side face

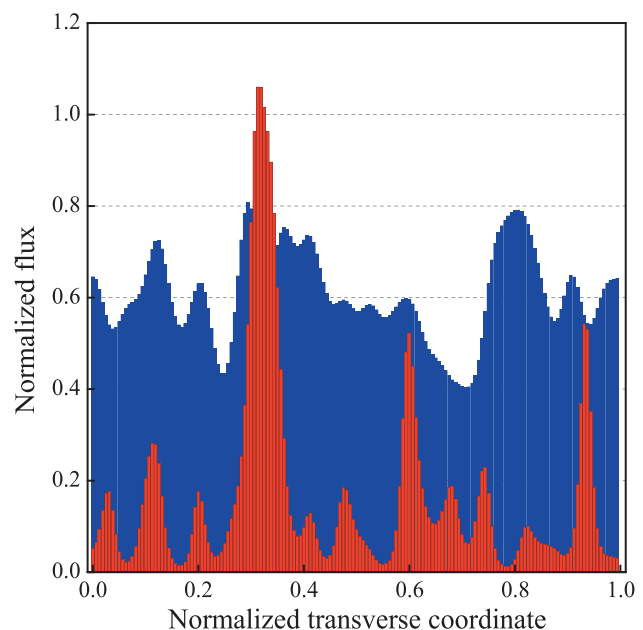


Table 2 Comparison of proportionality coefficients relating the mean flow rate to its variance: Monte Carlo simulation vs. Expression Eq. 17. The given values have the dimension of m^3/day

$\lambda \ m$	Monte Carlo simulation		Expression Eq. 17	
	$\langle k \rangle = 25 \ mD$	$\langle k \rangle = 50 \ mD$	$\langle k \rangle = 25 \ mD$	$\langle k \rangle = 50 \ mD$
200	0.05	0.09	0.05	0.1
300	0.09	0.18	0.08	0.19
350	0.12	0.25	0.13	0.26
400	0.16	0.33	0.17	0.34

here $f(a)$ is the probability density function of the parameters of the pulses and ν is the average number of pulses per unit time. When pulses are identical there is no integration over a and expressions Eq. 15 reduce to the well-known simplified expression.

$$\langle \xi \rangle = \nu \bar{a} \int_{-\infty}^{\infty} dx F(x) \quad \sigma_{\xi}^2 = \nu \bar{a}^2 \int_{-\infty}^{\infty} dx F^2(x) \quad (16)$$

Here $\bar{a}$ denotes the average amplitude of random pulses.

Expressions Eq. 16 can be used to calculate the flux variance, considering that, in the context of this study, time t_i corresponds to the coordinate of i -th pulse. If one assumes that the pulses have a Gaussian shape and their average width is equal to the correlation length, then the expression for the flux variance can be written as

$$\sigma_q^2 \sim \bar{A} \langle q \rangle \quad (17)$$

$$\bar{A} = \sqrt{2\pi} \frac{\langle k \rangle}{\mu} \frac{\Delta p}{\bar{L}} \lambda^2 \frac{s_z}{s_x} \quad (18)$$

here $\bar{L}$ denotes the average channel length. If to suppose for simplicity that the average channel length is approximately equal to the model size $\bar{L} \approx s_x$, the flux variance can be easily estimated using Eq. 17. The proportionality coefficient $\bar{A}$ between σ_q^2 and $\langle q \rangle$ in Eq. 17 can be directly compared to the slope obtained from the linear regression shown in the left plot of Fig. 2. The comparison results are presented in the Table 2 and there is good agreement between the analytical and numerical results.

To ensure the required accuracy, the numerical calculation of flux variance requires a larger number of realizations than that needed for computing the mean value. This number generally increases with either γ_k or λ . However, for $\gamma_k > \gamma_c$ and a given flux mean value $\langle q \rangle$ the variance can be evaluated using expression Eq. 17 without requiring additional computations.

4 Well inflow in stochastic porous media

In this section I focus on the problem of well inflow in reservoir with stochastic permeability. I consider the model with parameters listed in the Table 1 with a vertical well located at the center of the domain. The well is controlled through the specification of a constant bottom-hole pressure p_w , and flow is driven by the difference between p_w and side boundaries pressures p_0

$$p(r_w) = p_w \quad (19)$$

$$p(\Omega) = p_0$$

here r_w is well radius and Ω is the external model boundary. Equation 3 is solved numerically with subject to boundary condition Eq. 19 for each permeability realization. The well rate for i -th realization is found from the expression

$$q_i = W I_i(p_i - p_w) \quad (20)$$

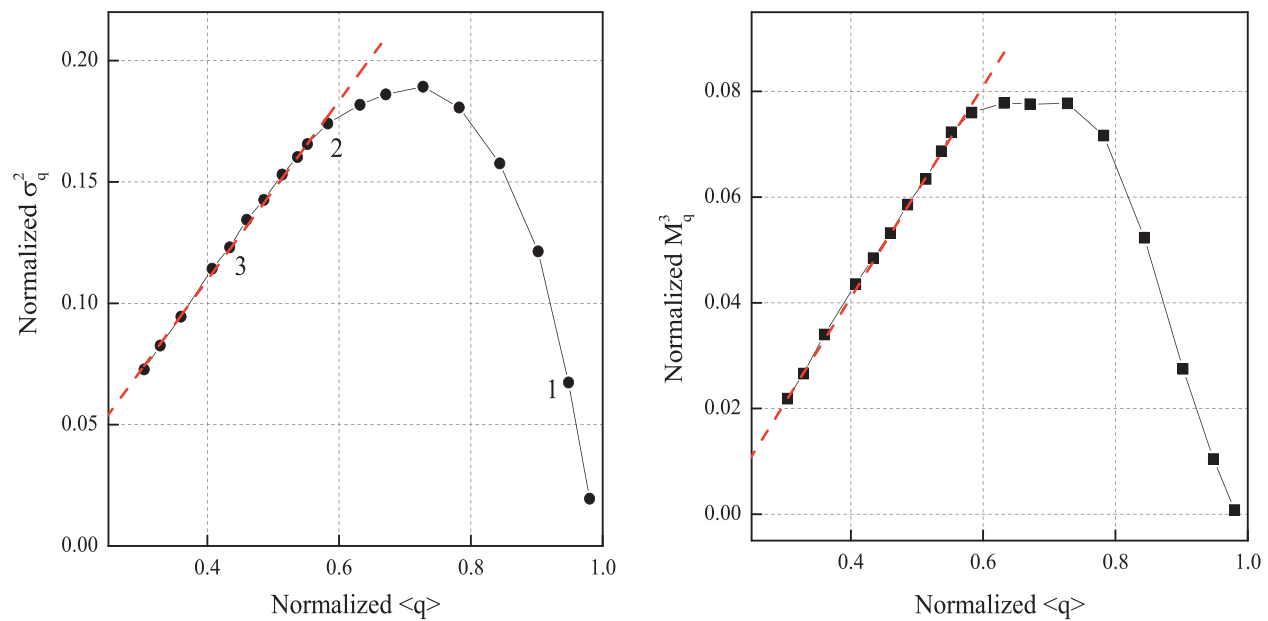


Fig. 6 Dependence of well rate variance (left) and the third (right) flux moment versus normalized rate mean value. Red dashed indicates linear fitting of rate variance for $\gamma_k > \gamma_c$. Point 1 on the left plot corresponds to $\gamma_k = 0.2$, points 2 and 3 correspond to values $\gamma_k = 2.0$ and $\gamma_k = 3.0$, respectively

where p_i and WI_i denote grid block pressure and well index. WI_i is given by Peaceman's expression.

$$WI_i = \frac{2\pi k_i \Delta z}{\ln \left(\frac{r_0}{r_w} \right)}$$

where r_0 is the equivalent well block radius, k_i is the permeability of well block in i -th realization. Using the formulas Eq. 5, the mean value and variance of the well flow rate are determined.

Figure 6 shows the dependencies of the second and third moments of the well rate on the mean value.

In the case of fluid inflow into a well, the majority of the pressure drop is localized within the near-wellbore zone, and the well production rate exhibits strong sensitivity to the permeability of the well block. Consequently,

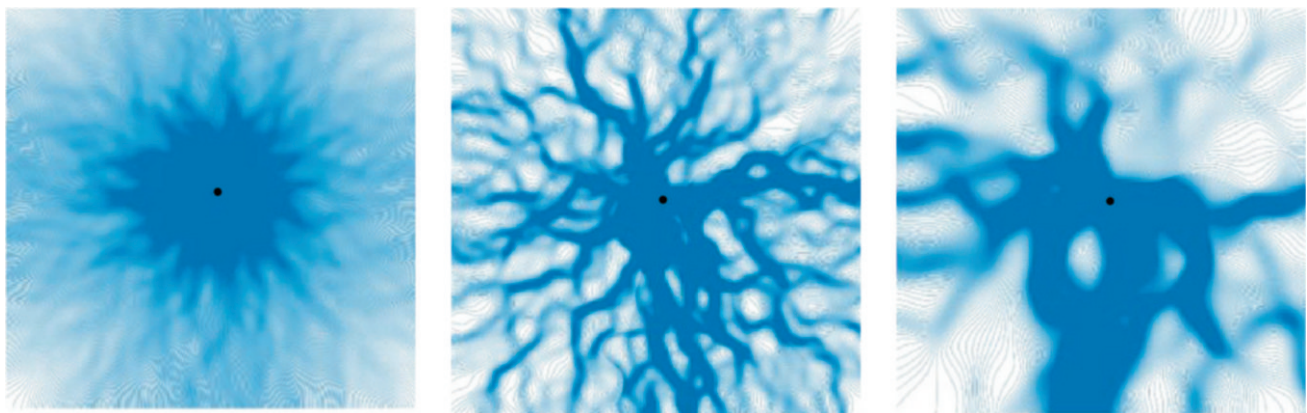
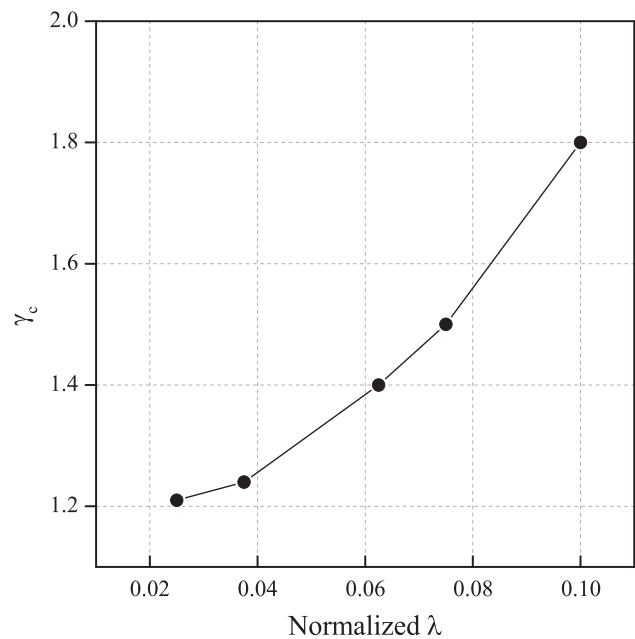


Fig. 7 Streamlines for different values of γ_k . The left plot illustrates the realization of streamlines for $\gamma_k = 0.2$ that corresponds to the point 1 on Fig. 6. The central and right plots depict streamlines for $\gamma_k = 2.0$ (point 2) and $\gamma_k = 3.0$ (point 3) correspondingly

Fig. 8 Dependence of γ_c on the normalized correlation length of the random permeability field



one might be expected that σ_q^2 would be proportional to well block permeability variance and increase with rising γ_k . However, as shown in Fig. 6, the behavior of σ_q^2 closely resembles that observed in Fig. 2.

Figure 7 presents streamlines for various values of γ_k . It can be seen that at small values γ_k the streamlines are evenly distributed across the drained area. As γ_k increases, their spatial distribution becomes non-uniform, resulting in the formation of conductive channels.

The value γ_c depends on the ratio λ/s_x . As the correlation length increases, γ_c shifts towards larger values of γ_k and conductive channels begin to form at larger γ_k . Figure 8 shows the dependence of γ_c on the correlation length and as can be seen the value of γ_k increases with the growth of λ .

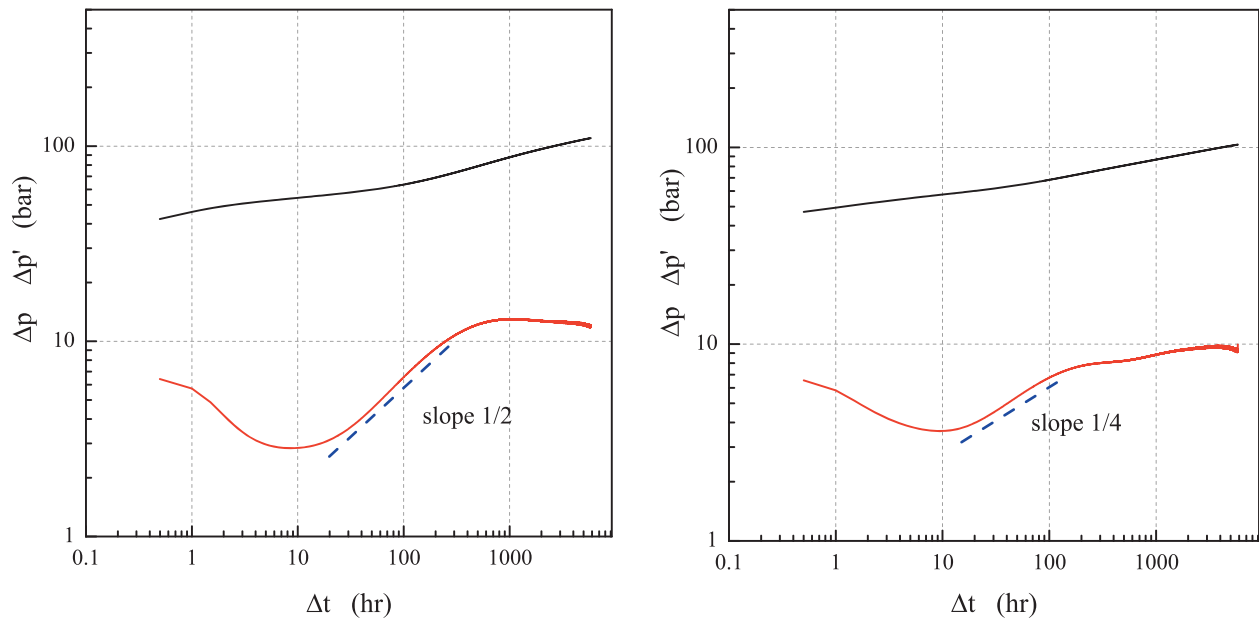


Fig. 9 Pressure build up log-log diagnostic plots for two realizations of a random permeability field with $\gamma_k = 3.0$ and $\lambda = 200 \text{ m}$. The logarithmic derivatives (red curves) indicate the presence of linear flow, corresponding to segments with slopes of $1/2$ and $1/4$

Figure 9 depicts pressure build up log-log diagnostic plots for two realizations of a random permeability field with $\gamma_k = 3.0$ and $\lambda = 200$ m. As can be seen, the logarithmic derivatives exhibit a linear region with slopes approximately equal to 1/2 (left plot) and 1/4 (right plot). These slopes indicate the presence of a linear or bilinear flow regime correspondingly. Similar behavior of pressure build up curves in reservoirs with stochastic permeability was observed in [19], under the condition $\gamma_k > 1.0$. The appearance of linear segments in the logarithmic derivative is evidently associated with the formation of discrete flow channels. However, it should not be forgotten that other factors such as the presence of a hydraulic fracture or the case of two parallel linear boundaries may also lead to the appearance of the same linear segments. Therefore, to ensure accurate interpretation of well test data, it is important to consider that a similar diagnostic plot may also arise in models of formations with stochastic permeability.

5 Two-phase well inflow in stochastic porous media

Up to this point, the analysis has focused on single-phase fluid flow, and it has been demonstrated that when $\gamma_k > \gamma_c$, the fluid transport occurs through a system of discrete channels. The formation of such channels is expected to have a significant impact on the oil displacement process. This section investigates the flow of a two-phase fluid (oil, water) in reservoirs with random permeability. This task is important in a number of applications in oil recovery. The governing equations for oil and water phases take the following form

$$-\nabla \cdot (\rho_\alpha \vec{v}_\alpha(\mathbf{r})) = \frac{\phi \partial(\rho_\alpha(\mathbf{r})s_\alpha(\mathbf{r}))}{\partial t} + q_\alpha \quad (21)$$

$$\vec{v}_\alpha(\mathbf{r}) = -\frac{\mathbf{k}(\mathbf{r})k_{r\alpha}}{\mu_\alpha} \nabla p_\alpha(\mathbf{r})$$

ρ_α and s_α are the density and saturation of phase α respectively. ϕ is the rock porosity and $k_{r\alpha}$ is the relative permeability of α -th phase which is a given and non-random function. q_α denotes the mass source term.

The model is initially saturated with oil, and an aquifer is specified at its boundaries. Unlike the single-phase scenario, the well is controlled by the liquid production rate. The relative permeability was adopted according to the Core correlation model. By solving equations Eq. 21, the temporal evolution of oil saturation and well water cut is determined for each realization of the permeability field.

Figure 10 shows oil saturation maps at different time steps corresponding to various values of γ_k and for the same correlation length. As illustrated in this figure, for $\gamma_k = 3.0$, water breakthrough into the well occurs significantly earlier compared to the case of a weakly heterogeneous reservoir $\gamma_k = 0.5$. The figure also shows that for $\gamma_k = 3$, there are regions with unrecovered oil reserves.

By following the same procedure as described in Section 2, an ensemble of water cut time profiles is generated, and P_{10} , P_{50} and P_{90} values for water breakthrough time are estimated.

Figure 11 (left) illustrates dependence of P_{90} normalized water breakthrough time on γ_k and the right plot depicts the well water cut as a function of dimensionless time. Time is normalized with respect to breakthrough time for uniform reservoir with permeability $\langle k \rangle$. In the right plot the red curve corresponds to $\gamma_k = 0.5$ and the black one corresponds to $\gamma_k = 3.0$. Thus, an increase in γ_k leads to a reduction in water breakthrough time. Similar results were obtained in [18], where it was demonstrated that heterogeneity accelerates water breakthrough in oil production.

Figure 12 shows the dependence of breakthrough time on the dimensionless correlation length. The correlation length is normalized to $s_x/2$, representing the distance from the well to the edges of the model. Unlike the dependence on γ_k , which follows a monotonic decreasing trend, the breakthrough time as a function of λ exhibits a distinct minimum.

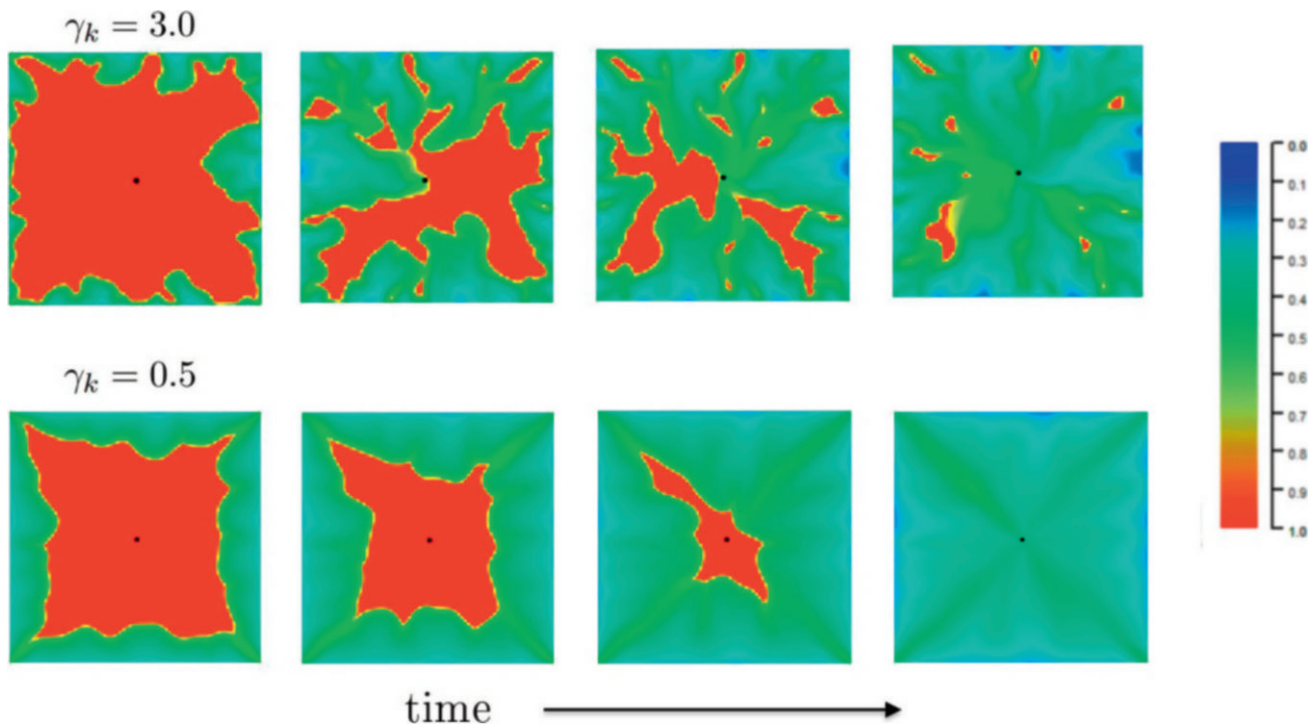


Fig. 10 Temporal evolution of the spatial oil saturation maps for $\gamma_k = 0.5$ and $\gamma_k = 3$

Figures 11 and 12 demonstrate that the statistical characteristics of the random permeability field significantly influence the timing of water breakthrough in the well. These characteristics also affect the overall efficiency of the waterflooding process, which relies on effective contact between the injected water and the displaced oil. The emergence of discrete flow channels can result in the development of stagnant zones where oil displacement is minimal, making standard water injection patterns suboptimal under such conditions.

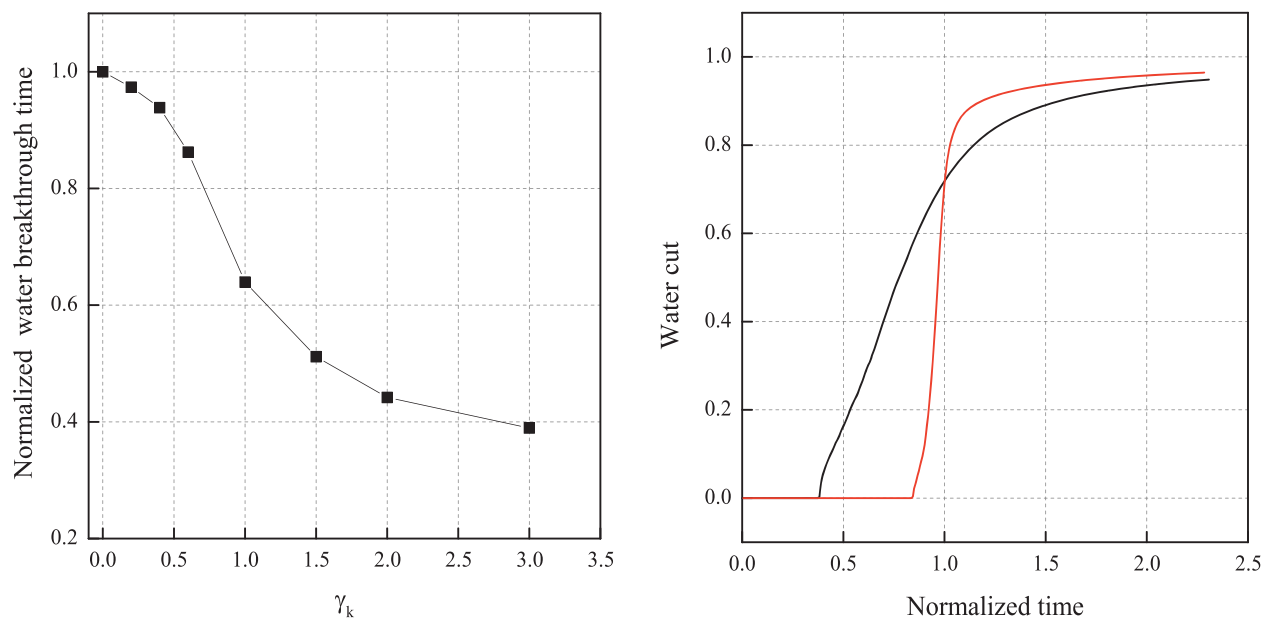


Fig. 11 Dependencies of P_{00} of water breakthrough time on the coefficient variation of permeability γ_k (left) and the water cut dependencies versus dimensionless time (right). The cases $\gamma_k = 0.5$ and $\gamma_k = 3.0$ are marked in black and red correspondingly

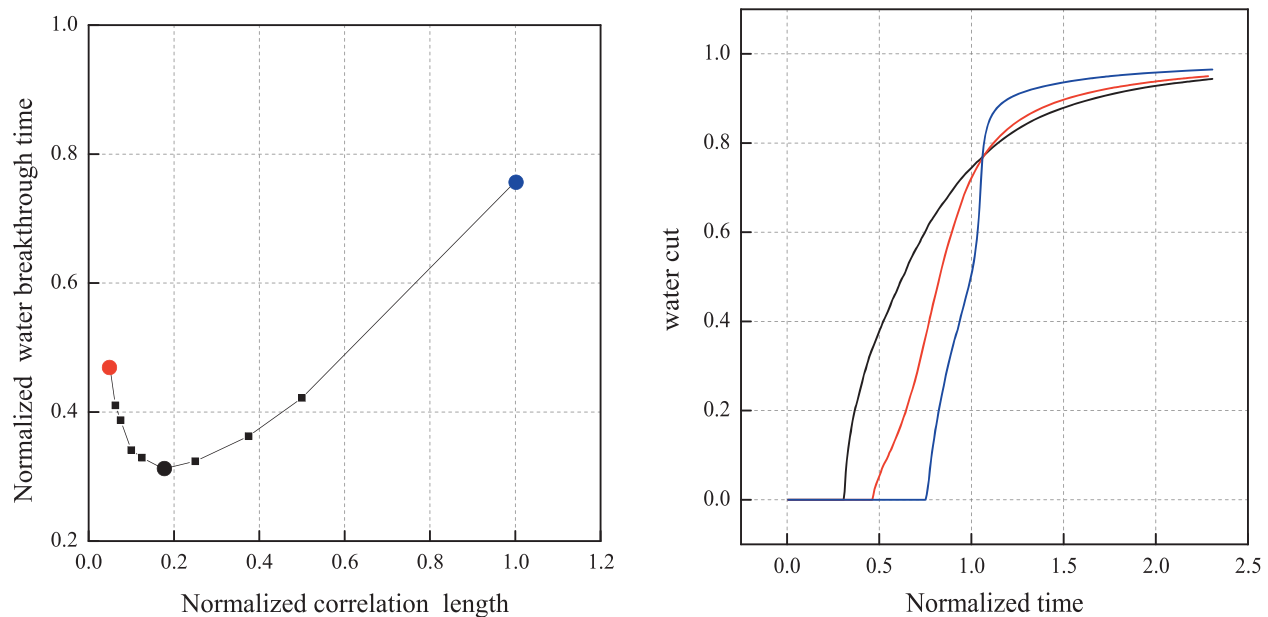


Fig. 12 Dependencies of P_{90} of water breakthrough time on the normalized correlation length (left) and the water cut dependencies versus dimensionless time (right). The case $\lambda = 200m$ is marked in red, while $\lambda = 700m$ and $\lambda = 4000m$ are marked in black and blue, respectively. The water breakthrough time is normalized by the breakthrough time for a homogeneous formation with permeability $\langle k \rangle$

6 Conclusion

In this paper, I address the problem of fluid flow in heterogeneous porous media with random permeability following a log-normal distribution. The flux variance was calculated as a function of the permeability's normalized standard deviation over a wide range of, spanning from 0.1 to 4.5. Two key scenarios were analyzed: fluid flow in a line drive reservoir and two-phase inflow into a well. The primary results of this study are as follows:

- The Monte Carlo method has been used to obtain the dependencies of the mean and flux variance on the standard deviation of the permeability field. The analytical expressions for flux variance presented in the literature do not match the results of numerical MCS at high values of the permeability fluctuation.
- It is shown that the dependence of flux variance σ_q^2 on γ_k has a maximum at $\gamma_k = \gamma_c$. The analytical expressions presented in the literature agree well with the Monte Carlo results for $\gamma_k < \gamma_c$ but exhibit both quantitative and qualitative discrepancies for $\gamma_k > \gamma_c$. The value of γ_c depends on the correlation length λ and shifts to higher values with an increase in λ . The dependence of γ_c on λ is presented.
- For $\gamma_k > \gamma_c$ the random flux follows the Poisson distribution, where the variance of the flux is directly proportional to its mean. This behavior is analogous to the phenomenon of shot noise observed in electric current flow. Within the specified range of γ_k values, a network of discrete channels develops in the reservoir, governed by the stochastic nature of the permeability field.
- By drawing an analogy with the phenomenon of shot noise, an expression for the flux variance was derived, demonstrating strong agreement with the results of Monte Carlo simulations. This expression allows for the calculation of flux variance for a given mean value without the need to compute additional realizations.
- The possibility of detecting the existence of channels from the analysis of pressure build up test is discussed.
- The emergence of a discrete network of conductive channels significantly affect the oil displacement process. Adjusting these values can significantly modify the dynamics of well water cut.

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Declarations

Ethical standards The manuscript complies with the ethical standards of the journal.

Conflict of Interests The author declare no conflict of interest.

References

- Noetinger, B.: The effective permeability of a heterogeneous porous medium. *Transp. Porous Media* **15**, 99–127 (1994)
- Shvidler, M.I.: Statistical hydrodynamic of porous media. Nedra, Moscow (1985)
- Bjerg, P.L., Hinsby, K., Christensen, T.H., Gravesen, P.: Spatial variability of hydraulic conductivity of an unconfined sandy aquifer determined by a mini slug test. *J. Hydrol.* **136**, 107–122 (1992)
- Rehfeldt, K.R., Boggs, J.M., Gelhar, L.W.: Field study of dispersion in a heterogeneous aquifer: 3. Geostatistical analysis of hydraulic conductivity. *Water Resour. Res.* **28**(12), 3309–3324 (1992)
- Dagan, G.: Stochastic modeling of groundwater flow by unconditional and conditional probabilities: 1. Conditional simulation and the direct problem. *Water Resour. Res.* **18**(4), 813–833 (1982)
- Noetinger, B., Gautier, Y.: Use of the Fourier-Laplace transform and of diagrammatical methods to interpret pumping tests heterogeneous reservoirs. *Adv. Water Resour.* **21**, 581–590 (1998)
- Novikov, A.V., Posvyanskii, D.V.: The use of Feynman diagrammatic approach for well test analysis in stochastic porous media: *Comput. Geosci.* **24**, 921–931 (2020)
- Zhang, D., Winter, C.L.: Solutions to statistical moment equations of groundwater flow in nonstationary, bounded, heterogeneous media. *Water Resour. Res.* **34**(3), 529–538 (1998)
- Li, L., Tchelepi, H.A., Zhang, D.: Perturbation-based moment equation approach for flow in heterogeneous porous media: applicability range and analysis of high-order terms. *J. Comp. Phys.* **188**, 296–317 (2003)
- Rytov, S.M.: Introduction to Statistical Radiophysics. Part 1: Random Processes. Science, Moscow (1976)
- Hassen A.E., Cushman J.H.: A Monte Carlo assessment of Eulerian flow and transport perturbation models. *Water Resour. Res.* **34**(5), (1998)
- Matheron, G.: Elements pour une theorie des milieux poreux. Masson et cie, Paris (1967)
- King, P.R.: The use of field theoretic methods for the study of flow in a heterogeneous porous medium. *J. Phys. A: Math. Gen.* **20**, 3935–3947 (1987)
- De Wit, A.: Correlation structure dependence of the effective permeability of heterogeneous porous media. *Phys. Fluids* **7**(11), 2553–2562 (1995)
- Kogan, Sh.: Electronic noise and fluctuations in solids to statistical radiophysics. Cambridge University Press, (1996)
- Zeldovich, Ya.B., Molchanov, S.A., Ruzmalkin, A.A., Sokolov, D.D.: Intermittency in random media. *Soviet Physic Uspechi* **30**(5), 921–931 (1987)
- Rubin, Y.: Stochastic modeling of macrodispersion in heterogeneous media. *Water Resour. Res.* **26**(1), 133–141 (1990)
- Zhang, D., Tchelepi, H.: Stochastic analysis of immiscible two-phase flow in heterogeneous media. *SPE J.* **4**(4), 380–388 (1999)
- Hamdi, H.: Well-test response in stochastic permeable media. *J. Petrol. Sci. Eng.* **119**, 169–184 (2014)

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Authors and Affiliations

Posvyanskii D.V.¹ 

✉ Posvyanskii D.V.
dimitrii.posvyanskii@aspentech.com

¹ AspenTech, Herzliya, Israel